

Limits

Def: Let $\phi(t)$ be a scalar function. Then

$$\lim_{t \rightarrow t_0} \phi(t) = \phi_0 \text{ if for every } \epsilon > 0$$

there exists a $\delta > 0$ s.t. $|\phi(t) - \phi_0| < \epsilon$
whenever $0 < |t - t_0| < \delta$

Limits of vector functions: Let $\vec{F}(t)$

be a vector valued function. Then

$$\lim_{t \rightarrow t_0} \vec{F}(t) = \vec{F}_0 \text{ if } \forall \epsilon > 0 \exists \text{ a } \delta > 0$$

$$|\vec{F}(t) - \vec{F}_0| < \epsilon \text{ whenever } 0 < |t - t_0| < \delta$$

Continuity

Scalar function: A scalar function

$\phi(t)$ is continuous at $t = t_0$ if

$$\lim_{t \rightarrow t_0} \phi(t) = \phi(t_0) \text{ i.e. } \forall \epsilon > 0 \exists \text{ a } \delta > 0$$

s.t. $|\phi(t) - \phi(t_0)| < \epsilon$ whenever

$$|t - t_0| < \delta$$

Vector function: A vector function $\vec{F}(t)$ is

said to be continuous at $t = t_0$ if

$$\lim_{t \rightarrow t_0} \vec{F}(t) = \vec{F}(t_0) \text{ i.e. } \forall \epsilon > 0 \exists \text{ a } \delta > 0$$

s.t. $\vec{F}(t) - \vec{F}(t_0) < \epsilon$ whenever $|t - t_0| < \delta$.

Ordinary Differentiation of Scalars:

Let $\phi(t)$ be a scalar field. Then the ordinary derivative of the scalar $\phi(t)$ is defined as

$$\frac{d\phi(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\phi(t + \Delta t) - \phi(t)}{\Delta t}$$

Ordinary Differentiation of vectors:

Let $\vec{F}(t)$ be a vector valued function. Then the ordinary derivative of the vector $\vec{F}(t)$ is defined as

$$\frac{d\vec{F}(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\vec{F}(t + \Delta t) - \vec{F}(t)}{\Delta t}$$